

$$\begin{aligned} \textcircled{9} \quad & \text{If } \tan \theta + \sec \theta = l \\ & \Rightarrow \frac{\sin \theta}{\cos \theta} + \frac{1}{\cos \theta} = l \\ & \Rightarrow \frac{1 + \sin \theta}{\cos \theta} = l \end{aligned}$$

Squaring both sides

$$\begin{aligned} & \left(\frac{1 + \sin \theta}{\cos \theta} \right)^2 = l^2 \\ \Rightarrow l^2 &= \frac{(1 + \sin \theta)^2}{\cos^2 \theta} \\ &= \frac{(1 + \sin \theta)^2}{1 - \sin^2 \theta} \\ &= \frac{(1 + \sin \theta)^2}{(1 - \sin \theta)(1 + \sin \theta)} \\ &= \frac{1 + \sin \theta}{1 - \sin \theta} \end{aligned}$$

$$\begin{aligned} \text{RHS} &= \frac{l^2 + 1}{2l} \\ &= \frac{\frac{1 + \sin \theta}{1 - \sin \theta} + 1}{2 \left(\frac{1 + \sin \theta}{\cos \theta} \right)} \\ &= \frac{1 + \sin \theta + 1 - \sin \theta}{2 \left(\frac{1 + \sin \theta}{\cos \theta} \right)} \\ &= \frac{2}{1 - \sin \theta} \times \frac{\cos \theta}{2(1 + \sin \theta)} \end{aligned}$$

$$\begin{aligned} &= \frac{\cos \theta}{1 - \sin^2 \theta} \\ &= \frac{\cos \theta}{\cos^2 \theta} \\ &= \frac{1}{\cos \theta} \\ &= \sec \theta \\ &= \text{LHS} \end{aligned}$$

$$\begin{aligned} \textcircled{10} \quad & p = \sin \theta + \cos \theta \dots \textcircled{i} \\ & q = \sec \theta + \csc \theta \\ &= \frac{1}{\cos \theta} + \frac{1}{\sin \theta} \\ & q = \frac{\sin \theta + \cos \theta}{\sin \theta \cos \theta} \dots \textcircled{ii} \end{aligned}$$

Squaring (i)

$$\begin{aligned} p^2 &= (\sin \theta + \cos \theta)^2 \\ &= \sin^2 \theta + \cos^2 \theta + 2 \sin \theta \cos \theta \\ p^2 &= 1 + 2 \sin \theta \cos \theta \dots \textcircled{iii} \end{aligned}$$

$$\begin{aligned} \text{LHS} &= q(p^2 - 1) \\ &= \frac{\sin \theta + \cos \theta}{\sin \theta \cos \theta} \left[1 + 2 \sin \theta \cos \theta - 1 \right] \\ &= \frac{\sin \theta + \cos \theta}{\cancel{\sin \theta \cos \theta}} \times 2 \sin \theta \cos \theta \\ &= 2p \quad (\text{using i}) \\ &= \text{RHS} \end{aligned}$$