

$$\textcircled{1} \operatorname{cosec} \theta + \cot \theta = p$$

$$\Rightarrow \frac{1}{\sin \theta} + \frac{\cos \theta}{\sin \theta} = p$$

$$\Rightarrow \frac{1 + \cos \theta}{\sin \theta} = p \dots$$

Squaring both sides

$$\frac{(1 + \cos \theta)^2}{\sin^2 \theta} = p^2$$

$$\begin{aligned} \Rightarrow p^2 &= \frac{(1 + \cos \theta)^2}{1 - \cos^2 \theta} \\ &= \frac{(1 + \cos \theta)^2}{(1 - \cos \theta)(1 + \cos \theta)} \\ &= \frac{1 + \cos \theta}{1 - \cos \theta} \end{aligned}$$

$$\text{RHS} = \frac{p^2 - 1}{p^2 + 1}$$

$$\begin{aligned} &= \frac{\frac{1 + \cos \theta}{1 - \cos \theta} - \frac{1}{1}}{\frac{1 + \cos \theta}{1 - \cos \theta} + \frac{1}{1}} \\ &= \frac{\frac{1 + \cos \theta - 1 + \cos \theta}{(1 - \cos \theta)}}{\frac{1 + \cos \theta + 1 - \cos \theta}{(1 - \cos \theta)}} \\ &= \frac{2 \cos \theta}{2} \end{aligned}$$

$$= \text{LHS}$$

$$\textcircled{2} \text{ LHS}$$

$$= \sqrt{\sec^2 \theta + \operatorname{cosec}^2 \theta}$$

$$= \sqrt{\frac{1}{\cos^2 \theta} + \frac{1}{\sin^2 \theta}}$$

$$= \sqrt{\frac{\sin^2 \theta + \cos^2 \theta}{\sin^2 \theta \cos^2 \theta}}$$

$$= \sqrt{\frac{1}{\sin^2 \theta \cos^2 \theta}}$$

$$= \frac{1}{\sin \theta \cos \theta}$$

$$= \frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta \cos \theta}$$

$$= \frac{\cancel{\sin^2 \theta}}{\cancel{\sin \theta} \cos \theta} + \frac{\cancel{\cos^2 \theta}}{\sin \theta \cancel{\cos \theta}}$$

$$= \tan \theta + \cot \theta$$

$$= \text{RHS}$$