

to prove - $\triangle BPA$ is isosceles

proof $\angle 2 = \angle 3$ [BX bis. $\angle ABC$]
 ... ①

$BA \parallel DP$

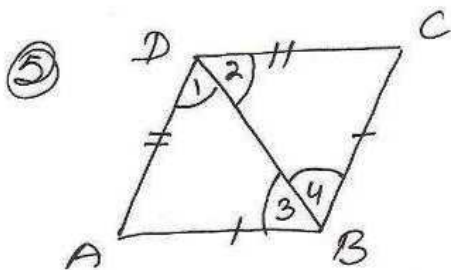
$\therefore \angle 3 = \angle 1$ (alternate interior angles)
 ... ②

From ①, ②

$$\angle 1 = \angle 2$$

$\Rightarrow BP = PA$
 [converse of isos. \triangle prop]

$\therefore \triangle BPA$ is isosceles



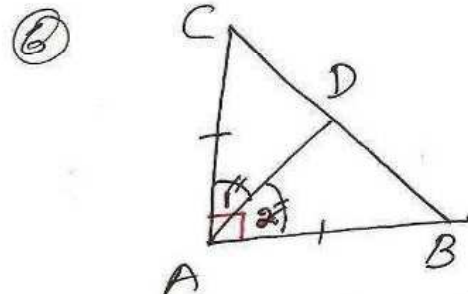
to show BD bisects $\angle B$ and $\angle D$

In $\triangle DAB$ and $\triangle DCB$
 $AB = CB$ (given)
 $AD = CD$ (do)
 $DB = DB$ (common)
 $\therefore \triangle DAB \cong \triangle DCB$ by SSS prop

$$\angle 1 = \angle 2 \text{ (cpct)}$$

$$\angle 3 = \angle 4$$

$\Rightarrow DB$ bisects $\angle D$ and $\angle B$



To prove $BC = 2AD$

proof In rt $\triangle ABC$
 $AB = AC$

$\Rightarrow BC$ is hypotenuse
 [\because hyp. is longest side]

$$\therefore \angle BAC = 90^\circ$$

In $\triangle CAD$ and $\triangle BAD$

$$AC = AB \text{ (given)}$$

$$\angle 1 = \angle 2$$

$$AD = AD \text{ (common)}$$

$\therefore \triangle CAD \cong \triangle BAD$ by SAS prop

$$\therefore CD = BD \text{ (cpct)}$$