

③ let $p(x) = px^2 + 5x + k$

$x-2$ is a factor of $p(x)$

$\therefore p(2) = 0$ by factor theorem

$$px^2 + 5x + k = 0$$

$$4p + 10 + k = 0$$

$$k = -4p - 10 \dots \textcircled{1}$$

$x - \frac{1}{2}$ is a factor of $p(x)$

$\therefore p\left(\frac{1}{2}\right) + 5 \times \frac{1}{2} + k = 0$

$$\frac{p}{4} + \frac{5}{2} + k = 0$$

$$\Rightarrow \frac{p + 10 + 4k}{4} = 0$$

$$\Rightarrow p + 10 + 4k = 0$$

$$\Rightarrow 4k = -p - 10$$

$$\Rightarrow k = -\frac{p+10}{4} \dots \textcircled{2}$$

From $\textcircled{1}, \textcircled{2}$

$$-4p - 10 = -\frac{p+10}{4}$$

$$\Rightarrow -16p - 40 = -p - 10$$

$$\Rightarrow -15p = -10 + 40$$

$$\Rightarrow p = -2$$

Sub $\textcircled{1}$

$$\begin{aligned} k &= -4(-2) - 10 \\ &= 8 - 10 \\ &= -2 \end{aligned}$$

$\therefore p = k$