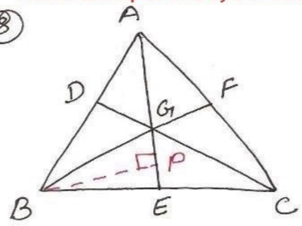
(8)



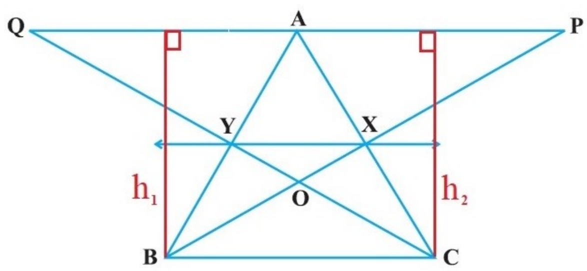
To Prove
$ar(AGB) = ar(AGC) = ar(BGC)$
$= \frac{1}{3} ar(ABC)$

Proof – $AG = \frac{2}{3} AE$ [Centroid divides the median in ratio 2:1]

$ar(\Delta AGB) = \frac{1}{2} \times AG \times BP$
$= \frac{1}{2} \times \frac{2}{3} AE \times BP$
$= \frac{2}{3} \times \frac{1}{2} \times AE \times BP$
$= \frac{2}{3}\ ar(\Delta ABE)$
$= \frac{2}{3} \times \frac{1}{2} ar(\Delta ABC)$ [Median divides a Δ into 2 Δs equal in area]
$= \frac{1}{3}\ ar(\Delta ABC)$

Similarly $ar(\Delta AGC) = ar(\Delta BGC) = \frac{1}{3} ar(\Delta ABC)$

(9)



Construction: draw $h_1 \perp QP$ and $h_2 \perp QP$

XY joins midpoints of sides AB and AC respectively of ΔABC
$\therefore YX \parallel BC$ (Mid point theorem)
But $QP \parallel BC$ (given)
$\therefore XY \parallel PQ$

In ΔABP, Y is midpoint of AB
$YX \parallel AP$
$\therefore$ X is midpoint of BP [converse of midpt. theorem]

In ΔABP, YX joins midpoints of sides AB and PB respectively
$\therefore YX = \frac{1}{2} AP \quad \ldots (i)$ [Midpoint theorem]

Sim $YX = \frac{1}{2} QA \ldots (ii)$

From (i), (ii)
$\frac{1}{2} AP = \frac{1}{2} QA$
$\Rightarrow AP = QA$

$$\frac{ar(\Delta ABP)}{ar(\Delta ACQ)} = \frac{\cancel{\frac{1}{2}} \times \cancel{QA} \times \cancel{h_1}}{\cancel{\frac{1}{2}} \times \cancel{AP} \times \cancel{h_2}}$$

[$\because AP = QA$]
[$\because h_1 = h_2$ distance between $\parallel$ sides]

$\Rightarrow ar(\Delta ABP) = ar(\Delta ACQ)$