ex 9.4, exemplar IX

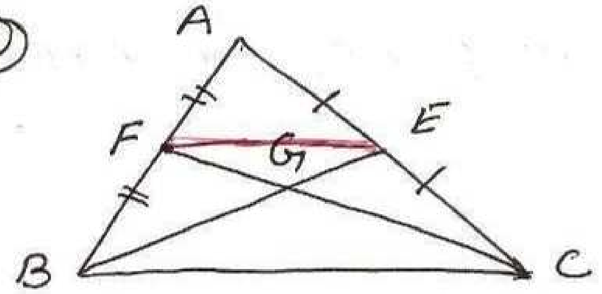
$ar(\triangle ADC) = ar(\triangle ABC)$ [diagonal divides a ||gm into 2 $\triangle$s equal in area]

$\Rightarrow ar(\triangle AOP) + ar(\square DPOC) = ar(\square BQOA) + ar(\triangle COQ)$

$\Rightarrow ar(\triangle COQ) + ar(\square DPOC) = ar(\square BQOA) + ar(\triangle AOP)$ [using ①]

$\Rightarrow ar(DPQC) = ar(APQB)$

③



to prove
$ar(\triangle GBC) = ar(\square AFGE)$

given - In figure BE and CF are medians to sides AB and AC resp.

Proof $ar(\triangle ABE) = \frac{1}{2} ar(\triangle ABC) \cdots$ ①
[Median divides a $\triangle$ into 2 $\triangle$s equal in area]

$ar(\triangle BFC) = \frac{1}{2} ar(\triangle ABC) \cdots$ ⑪ (do)

From ① and ⑪

$ar(\triangle ABE) = ar(\triangle BFC)$

$\Rightarrow ar(\square AFGC) + ar(\triangle BFG) = ar(\triangle GBC) + ar(\triangle CEG) \cdots$ ①

FE joins midpt. of sides AB and AC resp.

FE || BC [Mid pt. theorem]

$ar(\triangle FBC) = ar(\triangle ECB)$ [$\triangle$s on same base and between same || lines]

$ar(\triangle FBC) - ar(\triangle GBC) = ar(\triangle ECB) - ar(\triangle GBC)$

$\Rightarrow ar(\triangle BFG) = ar(\triangle CEG) \cdots$ ⑪

From ①, ⑪

$ar(\square AFGC) + ar(\cancel{\triangle} CEG) = ar(\triangle GBC) + ar(\cancel{\triangle} CEG)$

$\Rightarrow ar(\triangle GBC) = ar(\square AFGE)$