Chords AC and BD of a circle bisect each other. Prove AC and BD are diameters and quadrilateral ABCD is a rectangle.

to prove   AC and BD are diameters
□ ABCD is a rectangle

proof   In △ AOB and △COD

$$OA = OC \qquad (given)$$
$$\angle AOB = \angle COD \qquad (vert.\ opp.\ angles)$$
$$OB = OD \qquad (given)$$

△ AOB ≅ △ COD by SAS property

Sim.
$$AB = CD \qquad \Rightarrow \widehat{AB} = \widehat{CD} \cdots ①$$
$$AD = BC \qquad \Rightarrow \widehat{DA} = \widehat{BC} \cdots ⑪$$

① + ⑪
$$\widehat{AB} + \widehat{DA} = \widehat{CD} + \widehat{BC}$$

$$\Rightarrow \widehat{DB} = \widehat{BD}$$
⇒ BD divides ⊙ into 2 Semi ⊙s
∴ BD is diameter of ⊙
Sim.   AC is diameter of ⊙
∴ AC = BD
and AB = CD, AD = BC (proved)
∴ □ ABCD is a rectangle