



Let the vertex of an angle ABC be located outside a circle and the sides of the angle intersect equal chords AD and CE with the circle. Prove that angle ABC is equal to half the difference of the angles subtended by the chords AC and DE at the centre.

to prove $\angle B = \frac{1}{2} (\angle AOC - \angle DOE)$

proof $AD = CE$ (given)

$\therefore \angle 1 = \angle 2$ (equal chords subtend equal angles at the centre of \odot)

In $\triangle OAD$, $OA = OD$ (radii of same \odot)

$\Rightarrow \angle 3 = \angle 4$ (isosceles \triangle property)

let $\angle 3 = \angle 4 = a^\circ$

In $\triangle OCE$, $OC = OE$ (radii of same \odot)

$\Rightarrow \angle 5 = \angle 6$ (isosceles \triangle property)

let $\angle 5 = \angle 6 = b^\circ$

$\angle 1 + \angle 3 + \angle 4 = \angle 2 + \angle 5 + \angle 6 = 180^\circ$ (angle sum prop. of \triangle)

$\angle 1 + a^\circ + a^\circ = \angle 2 + b^\circ + b^\circ$ ($\because \angle 1 = \angle 2$)

$\Rightarrow 2a = 2b$

$\Rightarrow a = b$