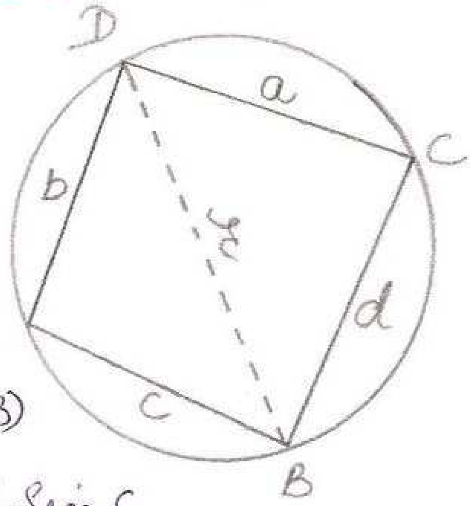
Area of Cyclic Quadrilateral by Brahmagupta's Formula

[For CBSE Class IX]



[cyclic □ - a □ whose all 4 vertices lie on a circle.]

Proof

$ar(\square ABCD) = ar(\Delta ABD) + ar(\Delta CDB)$

$= \frac{1}{2} bc \sin A + \frac{1}{2} ad \sin C$

[area of a Δ = $\frac{1}{2}$ product of 2 given sides and sine of angle between them]

let $ar(\square ABCD) = x$

$x = \frac{1}{2} bc \sin A + \frac{1}{2} ad \sin A \quad \ldots (1)$

[□ABCD is cyclic. opposite angles of a cyclic □ are supplementary $\therefore A + C = 180°$

$\Rightarrow A = 180° - C \quad \ldots (11)$

now $\sin A = \sin(180° - C)$ (using eqn ii)

$= \sin C$ [$\sin(180 - \theta) = \sin\theta$]

$\Rightarrow x = \frac{1}{2}(bc + ad)\sin A$ [taking $\frac{1}{2}\sin A$ common from RHS]

Squaring both sides

$x^2 = \frac{1}{4}(bc + ad)^2 \sin^2 A$

$\Rightarrow 4x^2 = (bc + ad)^2 (1 - \cos^2 A)$ [$\because \sin^2 A = 1 - \cos^2 A$]

[using trigonometric identity $\sin^2\theta + \cos^2\theta = 1$]