

Area of Cyclic Quadrilateral by Brahmagupta's Formula

[For CBSE Class IX]

$$\begin{aligned} \Rightarrow 16x^2 &= [2(bc+ad) + b^2 + c^2 - a^2 - d^2][2(bc+ad) - b^2 - c^2 + a^2 + d^2] \\ &= [b^2 + c^2 + 2bc - (a^2 + d^2 - 2ad)][a^2 + d^2 + 2ad - (b^2 + c^2 - 2bc)] \\ &= [(b+c)^2 - (a-d)^2][(a+d)^2 - (b-c)^2] \\ &\quad [\because A^2 + B^2 \pm 2AB = (A \pm B)^2] \end{aligned}$$

$$\begin{aligned} &= (b+c+a-d)(b+c-a+d)(a+d+b-c)(a+d-b+c) \\ &\quad [\text{using } a^2 - b^2 = (a+b)(a-b)] \end{aligned}$$

$$\begin{aligned} &= (a+b+c+d-2d)(a+b+c+d-2a)(a+b+c+d-2c)(a+b+c+d-2b) \\ &\quad [\text{adding and subtracting} \\ &\quad \quad [d, a, c, b \text{ respectively}]] \end{aligned}$$

$$\begin{aligned} &= (P-2d)(P-2a)(P-2c)(P-2b) \\ &\quad [\text{Perimeter } (P) = a+b+c+d] \end{aligned}$$

$$\begin{aligned} &= \left(\frac{2P}{2} - 2d\right)\left(\frac{2P}{2} - 2a\right)\left(\frac{2P}{2} - 2c\right)\left(\frac{2P}{2} - 2b\right) \\ &\quad [\text{Multiplying and dividing} \\ &\quad \quad [P \text{ by } 2]] \end{aligned}$$

$$\begin{aligned} &= 2^4 \left(\frac{P}{2} - d\right)\left(\frac{P}{2} - a\right)\left(\frac{P}{2} - c\right)\left(\frac{P}{2} - b\right) \\ &\quad [\text{taking 2 common}] \end{aligned}$$

$$\begin{aligned} \Rightarrow 16x^2 &= 16(s-a)(s-b)(s-c)(s-d) \\ &\quad [\text{Semiperimeter } (s) = \frac{P}{2}] \end{aligned}$$

$$\begin{aligned} \Rightarrow x &= \sqrt{(s-a)(s-b)(s-c)(s-d)} \\ &\quad \text{where } s \text{ is semiperimeter} = \frac{a+b+c+d}{2} \end{aligned}$$