$\Rightarrow 4x^2 = (bc+ad)^2 - (bc+ad)^2 \cos^2 A \quad \dots (iii)$

In $\triangle ABD$

$r^2 = b^2 + c^2 - 2bc \cos A \quad \dots (iv)$

In $\triangle CBD$

$r^2 = a^2 + d^2 - 2ad \cos C \quad \dots (v)$

[using cosine formula]

From (iv) and (v)

$b^2 + c^2 - 2bc \cos A = a^2 + d^2 - 2ad \cos C$

$b^2 + c^2 - 2bc \cos A = a^2 + d^2 - 2ad \cos(180 - A)$

[$\because$ $\square ABCD$ is cyclic
$\therefore A + C = 180$
$\Rightarrow C = 180 - A$]

$b^2 + c^2 - 2bc \cos A = a^2 + d^2 + 2ad \cos A$

[$\because \cos(180° - \theta) = -\cos\theta$]

$\Rightarrow b^2 + c^2 - a^2 - d^2 = 2bc \cos A + 2ad \cos A$

$\Rightarrow b^2 + c^2 - a^2 - d^2 = 2(bc + ad) \cos A$

[taking $2 \cos A$ common from R.H.S.]

Squaring both sides

$(b^2 + c^2 - a^2 - d^2)^2 = 4(bc+ad)^2 \cos^2 A$

$\Rightarrow \frac{1}{4}(b^2 + c^2 - a^2 - d^2)^2 = (bc+ad)^2 \cos^2 A \quad \dots (vi)$

From (iii) and (vi)

$4x^2 = (bc+ad)^2 - \frac{1}{4}(b^2 + c^2 - a^2 - d^2)^2$

$16x^2 = 4(bc+ad)^2 - (b^2 + c^2 - a^2 - d^2)^2$

[Multiplying both sides by 4]

$\Rightarrow 16x^2 = [2(bc+ad)]^2 - (b^2 + c^2 - a^2 - d^2)^2$