

ex. 1.3 exemplar problems E-CHI-P14

$$11 \text{ (i)} \quad \frac{5+2\sqrt{3}}{7+4\sqrt{3}} = a-6\sqrt{3}$$

$$\begin{aligned} \Rightarrow a-6\sqrt{3} &= \frac{5+2\sqrt{3}}{7+4\sqrt{3}} \times \frac{7-4\sqrt{3}}{7-4\sqrt{3}} \\ &= \frac{35-20\sqrt{3}+14\sqrt{3}-8 \times 3}{7^2-(4\sqrt{3})^2} \\ &= \frac{35-6\sqrt{3}-24}{49-48} \end{aligned}$$

$$\Rightarrow a-6\sqrt{3} = 11-6\sqrt{3}$$

equating rat. parts on both sides

$$a = 11$$

$$\begin{aligned} 11 \text{ (ii)} \quad a\sqrt{5} - \frac{19}{11} &= \frac{3-\sqrt{5}}{3+2\sqrt{5}} \\ &= \frac{3-\sqrt{5}}{3+2\sqrt{5}} \times \frac{3-2\sqrt{5}}{3-2\sqrt{5}} \\ &= \frac{9-6\sqrt{5}-3\sqrt{5}+2 \times 5}{3^2-(2\sqrt{5})^2} \\ &= \frac{19-9\sqrt{5}}{9-20} \\ &= \frac{19-9\sqrt{5}}{-11} \end{aligned}$$

$$\Rightarrow a\sqrt{5} - \frac{19}{11} = -\frac{19}{11} + \frac{9\sqrt{5}}{11}$$

comp. var. parts

$$a = \frac{9}{11}$$

$$\begin{aligned} 11 \text{ (iii)} \quad 2-b\sqrt{6} &= \frac{\sqrt{2}+\sqrt{3}}{3\sqrt{2}-2\sqrt{3}} \\ &= \frac{\sqrt{2}+\sqrt{3}}{3\sqrt{2}-2\sqrt{3}} \times \frac{3\sqrt{2}+2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}} \\ &= \frac{3 \times 2 + 2\sqrt{6} + 3\sqrt{6} + 2 \times 3}{(3\sqrt{2})^2 - (2\sqrt{3})^2} \\ &= \frac{6+5\sqrt{6}+6}{9 \times 2 - 4 \times 3} \\ &= \frac{12+5\sqrt{6}}{18-12} \\ &= \frac{12+5\sqrt{6}}{6} \end{aligned}$$

$$2-b\sqrt{6} = \frac{12}{6} + \frac{5}{6}\sqrt{6}$$

comp. var. parts

$$b = -\frac{5}{6}$$

$$\begin{aligned} \text{(iv)} \quad a + \frac{7}{11}\sqrt{5}b &= \frac{7+\sqrt{5}}{7-\sqrt{5}} - \frac{7-\sqrt{5}}{7+\sqrt{5}} \\ &= \frac{(7+\sqrt{5})^2 - (7-\sqrt{5})^2}{7^2 - (\sqrt{5})^2} \\ &= \frac{49+5+14\sqrt{5} - 49-5+14\sqrt{5}}{49-5} \\ &= \frac{28\sqrt{5}}{44} \end{aligned}$$

$$\Rightarrow a + \frac{7}{11}\sqrt{5}b = 0 + \frac{7}{11}\sqrt{5}$$

comparing rat and var. parts

$$a = 0, b = 1$$