

$$\frac{p}{q} - \frac{2q}{p} = \sqrt{n+1} + \sqrt{n-1} - \sqrt{n+1} + \sqrt{n-1} \quad \text{(i) - (ii)}$$

$$\Rightarrow \frac{p^2 - 2q^2}{pq} = 2\sqrt{n-1}$$

$$\Rightarrow \frac{p^2 - 2q^2}{2pq} = \sqrt{n-1}$$

LHS = $\frac{p^2 - 2q^2}{2pq}$ is rational [num. is integer
by closure prop.
of integers for
mul. and sub.
den is integer]

$\therefore$ RHS = $\sqrt{n-1}$ is rational

Similarly $\sqrt{n+1}$ is rational

$\Rightarrow n+1, n-1$ are perfect squares

$$\text{difference} = n+1 - n-1$$

$$= 2$$

which is not possible,

$\therefore$ difference of two perfect squares is

at least 3

$\therefore$ our supposition is wrong

[$\sqrt{\text{Perfect Square}}$
is rational]