(1) Show that there is no positive integer n for which $\sqrt{n-1}+\sqrt{n+1}$ is rational

suppose there exists a +ve number n for which $\sqrt{n+1}+\sqrt{n-1}$ is rational

let $\sqrt{n+1}+\sqrt{n-1} = \frac{p}{q}$ ...(1) where p, q are +ve integers

applying invertendo

$$\frac{q}{p} = \frac{1}{\sqrt{n+1}+\sqrt{n-1}} \times \frac{\sqrt{n+1}-\sqrt{n-1}}{\sqrt{n+1}-\sqrt{n-1}}$$

$$= \frac{\sqrt{n+1}-\sqrt{n-1}}{(\sqrt{n+1})^2-(\sqrt{n-1})^2}$$

$$= \frac{\sqrt{n+1}-\sqrt{n-1}}{\cancel{n}+1-\cancel{n}+1}$$

$$= \frac{\sqrt{n+1}-\sqrt{n-1}}{2}$$

$$\Rightarrow \frac{2q}{p} = \sqrt{n+1}-\sqrt{n-1} \quad \ldots (11)$$

(1) + (11)

$$\frac{p}{q}+\frac{2q}{p} = \sqrt{n+1}+\cancel{\sqrt{n-1}}+\sqrt{n+1}-\cancel{\sqrt{n-1}}$$

$$\Rightarrow \frac{p^2+2q^2}{pq} = 2\sqrt{n+1}$$

$$\Rightarrow \frac{p^2+2q^2}{2pq} = \sqrt{n+1}$$