Let a, b, c, d be positive rationals such that $a + \sqrt{b} = c + \sqrt{d}$ then either $a = c$ and $b = d$ or b and d are squares of rationals

If $a = c$, $\cancel{a} + \sqrt{b} = \cancel{c} + \sqrt{d}$

$\Rightarrow \sqrt{b} = \sqrt{d}$

$\Rightarrow b = d$

If $a \neq c$ $\Rightarrow a = c + x$ ...(i) where x is +ve Q

$a + \sqrt{b} = c + \sqrt{d}$

$\cancel{c} + x + \sqrt{b} = \cancel{c} + \sqrt{d}$

$\Rightarrow x + \sqrt{b} = \sqrt{d}$ ... (1)

Squaring both sides

$x^2 + b + 2\sqrt{b}\,x = d$

$\Rightarrow d - x^2 - b = 2\sqrt{b}\,x$

$$\Rightarrow \sqrt{b} = \frac{d - x^2 - b}{2x}$$

$\Rightarrow \sqrt{b}$ is rational $\because$ RHS is Q

$\Rightarrow b$ is square of Q. no.

using (1)

$x + \sqrt{b} = \sqrt{d}$

LHS is Q

$\therefore$ RHS $= \sqrt{d}$ is Q

$\Rightarrow d$ is square of rational no.

$\therefore$ either $a = c$, $b = d$ or b, d are squares of Qs