

$$\begin{aligned} \textcircled{3} \quad a_8 &= \frac{1}{2} a_2 \\ \Rightarrow a + 7d &= \frac{1}{2} (a + d) \\ \Rightarrow 2a + 14d &= a + d \\ \Rightarrow a &= -13d \dots \textcircled{i} \end{aligned}$$

$$\begin{aligned} a_{11} &= \frac{1}{3} a_4 + 1 \\ a + 10d &= \frac{a + 3d}{3} + \frac{1}{1} \\ a + 10d &= \frac{a + 3d + 3}{3} \\ \Rightarrow 3a + 30d &= a + 3d + 3 \\ \Rightarrow 2a + 27d &= 3 \end{aligned}$$

using \textcircled{i}

$$\begin{aligned} 2(-13d) + 27d &= 3 \\ \Rightarrow d &= 3 \end{aligned}$$

Sub $\textcircled{1}$

$$\begin{aligned} a &= -13 \times 3 \\ &= -39 \end{aligned}$$

$$\begin{aligned} a_{15} &= a + 14d \\ &= -39 + 14 \times 3 \\ &= -39 + 42 \\ &= 3 \end{aligned}$$

$$\begin{aligned} \textcircled{4} \quad n &= 37 \\ \text{Middlemost term} &= \frac{n+1}{2} \\ &= \frac{38}{2} \\ &= 19^{\text{th}} \end{aligned}$$

Three middlemost terms are $18^{\text{th}}, 19^{\text{th}}, 20^{\text{th}}$

$$\begin{aligned} a_{18} + a_{19} + a_{20} &= 225 \\ a + 17d + a + 18d + a + 19d &= 225 \\ \Rightarrow 3a + 54d &= 225 \\ (\div 3) \quad a + 18d &= 75 \dots \textcircled{1} \end{aligned}$$

$$\begin{aligned} a_{35} + a_{36} + a_{37} &= 429 \\ \Rightarrow a + 34d + a + 35d + a + 36d &= 429 \\ \Rightarrow 3a + 105d &= 429 \\ (\div 3) \quad a + 35d &= 143 \dots \textcircled{ii} \end{aligned}$$

$$\begin{aligned} \textcircled{ii} - \textcircled{1} \\ \cancel{a} + 35d - \cancel{a} - 18d &= 143 - 75 \\ \Rightarrow 17d &= 68 \\ \Rightarrow d &= 4 \end{aligned}$$

Sub $\textcircled{1}$

$$\begin{aligned} a + 72 &= 75 \\ \Rightarrow a &= 3 \end{aligned}$$

\therefore required AP is
3, 7, 11, 15, ...